Radar Systems Engineering
Lecture 4
The Radar Equation

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The Radar Range Equation Connects:

1. **Target** Properties - e.g. Target Reflectivity (radar cross section)
2. **Radar** Characteristics - e.g. Transmitter Power, Antenna Aperture
3. Distance between **Target** and **Radar** - e.g. Range
4. Properties of the **Medium** - e.g. Atmospheric Attenuation.
Outline

- Introduction
- Introduction to Radar Equation
- Surveillance Form of Radar Equation
- Radar Equation for Rain Clutter
- Radar Losses
- Examples
- Summary
Key Radar Functions

• Detection
  – Illuminate selected area with enough energy to detect targets of interest
• Measure target observables
  – Measure range, Doppler and angular position of detected targets
• Track
  – Correlate successive target detections as coming from same object and refine state vector of target
• Identification
  – Determine what target is - Is it a threat?
• Handover
  – Pass the target on to;
    Missile interceptor
    Data Collection function
    Air Traffic Controller / Operator
Radar Range Equation

Power density from uniformly radiating antenna transmitting spherical wave

\[ \frac{P_t}{4\pi R^2} \]

- \( P_t \) = peak transmitter power
- \( R \) = distance from radar

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Radar Range Equation (continued)

Power density from isotropic antenna

\[
\frac{P_t}{4\pi R^2}
\]

Power density from directive antenna

\[
\frac{P_t G_t}{4\pi R^2}
\]

Gain is the radiation intensity of the antenna in a given direction over that of an isotropic (uniformly radiating) source

\[
G_t = \frac{4\pi A}{\lambda^2}
\]

\[P_t = \text{peak transmitter power}\]
\[R = \text{distance from radar}\]
\[G_t = \text{transmit gain}\]

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Radar Cross Section (RCS or $\sigma$) is a measure of the energy that a radar target intercepts and scatters back toward the radar.

\[
\sigma = \text{radar cross section units (meters)}^2
\]

Power of reflected signal at target:
\[
P_{\text{t}} G_{\text{t}} \sigma \over 4 \pi R^2
\]

Power density of reflected signal at the radar:
\[
P_{\text{t}} G_{\text{t}} \sigma \over 4 \pi R^2 \cdot 4 \pi R^2
\]

Power density of reflected signal falls off as $(1/R^2)$.
Radar Range Equation (continued)

Power density of reflected signal at radar

\[
\frac{P_t G_t}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2}
\]

The received power = the power density at the radar times the area of the receiving antenna

Power of reflected signal from target and received by radar

\[
P_r = \frac{P_t G_t}{4\pi R^2} \cdot \frac{\sigma A_e}{4\pi R^2}
\]

\(P_r\) = power received

\(A_e\) = effective area of receiving antenna

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Sources of Noise Received by Radar

- The total effect of the different noise sources is represented by a single noise source at the antenna output terminal.

- The noise power at the receiver is: \( N = k B_n T_s \)

**Sources of Noise**

- Solar Noise
- Galactic Noise
- Atmospheric Noise
- Man Made Interference (Radio Stations, Radars, etc)
- Ground Noise
- Solar Noise
- Galactic Noise
- Man Made Interference (Receiver, waveguide, and duplexer noise)
- Ground Noise

Equations:

- \( k \) = Boltzmann constant
  - \( = 1.38 \times 10^{-23} \text{ joules} / \text{ deg }^\circ \text{K} \)
- \( T_s \) = System Noise Temperature
- \( B_n \) = Noise bandwidth of receiver

Noise from Many Sources Competes with the Target Echo
Radar Range Equation (continued)

Signal Power reflected from target and received by radar

\[ P_r = \frac{P_t G_t}{4\pi R^2} \frac{\sigma A_e}{4\pi R^2} \]

Average Noise Power

\[ N = k B_n T_s \]

Signal to Noise Ratio

\[ \frac{S}{N} = \frac{P_r}{N} \]

Assumptions:

\[ G = G_r = G_t \]

\[ L = \text{Total System Losses} \]

\[ T_0 = 290^\circ K \]

Signal to Noise Ratio (S/N or SNR) is the standard measure of a radar’s ability to detect a given target at a given range from the radar

“\[ S/N = 13 \text{ dB on a } 1 \text{ m}^2 \text{ target at a range of 1000 km} \]”

radar cross section of target

Courtesy of MIT Lincoln Laboratory
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System Noise Temperature

The System Noise Temperature, $T_s$, is divided into 3 components:

$$T_s = T_a + T_r + L_r T_e$$

- $T_a$ is the contribution from the antenna
- $T_r$ is the contribution from the RF components between the antenna and the receiver
- $L_r$ is loss of the input RF components (natural units)
- $T_e$ is temperature of the receiver

The 3 temperature components can be broken down further:

$$T_a = (0.88 T_{sky} - 254)/(L_a + 290)$$

$$T_r = T_{tr} (L_r - 1) \quad \text{and} \quad T_e = T_o (F_n - 1)$$

Where:

- $T_{sky}$ is the apparent temperature of the sky (from graph)
- $L_a$ is the dissipative loss within the antenna (natural units)
- $T_{tr}$ is physical temperature of the RF components
- $F_n$ is the noise factor of the receiver (natural units)
- $T_o$ is the reference temperature of $290^\circ K$

Note that all temperature quantities are in units of $^\circ K$
Noise Temperature vs. Frequency

- The data on this graph takes into account the following effects:
  - Galactic noise, cosmic blackbody radiation, solar noise, and atmospheric noise due to the troposphere

(Adapted from Blake, Reference 5, p 170)
Outline

• Introduction

• Introduction to Radar Equation

• Surveillance Form of Radar Equation

• Radar Equation for Rain Clutter

• Radar Losses

• Examples

• Summary
Track Radar Equation

\[
\frac{S}{N} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_s B_n L}
\]

- When the location of a target is known and the antenna is pointed toward the target.

Track Example

Courtesy of MIT Lincoln Laboratory
Used with Permission
Development of Search Radar Equation

Track Radar Equation

\[
\frac{S}{N} = \frac{P_t \cdot G^2 \cdot \lambda^2 \cdot \sigma}{(4\pi)^3 R^4 k T_s B_n L}
\]

- When the location of a target is known and the antenna is pointed toward the target.

Search Radar Equation

\[
\frac{S}{N} = \frac{P_{av} \cdot A_e \cdot t_s \cdot \sigma}{4\pi \cdot \Omega \cdot R^4 \cdot k T_s \cdot L}
\]

- When the target’s location is unknown, and the radar has to search a large angular region to find it.

Where:
- \( P_{av} \) = average power
- \( \Omega \) = solid angle searched
- \( t_s \) = scan time for \( \Omega \)
- \( A_e \) = antenna area

Search Volume

Search Example

Courtesy of MIT Lincoln Laboratory
Used with Permission
Search Radar Range Equation

\[ \frac{S}{N} = \frac{P_{av} A_e t_s \sigma}{4\pi \Omega R^4 k T_s L} \]

Re-write as:

\[ f (\text{design parameters}) = g (\text{performance parameters}) \]

- Angular coverage
- Range coverage
- Measurement quality
- Time required
- Target size

Courtesy of MIT Lincoln Laboratory
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Scaling of Radar Equation

\[
\frac{S}{N} = \frac{P_{av} A_e t_s \sigma}{4 \pi \Omega R^4 k T_s L} \quad \rightarrow \quad P_{av} = \frac{4\pi R^4 \Omega k T_s L (S/N)}{A_e t_s \sigma}
\]

• Power required is:
  – Independent of wavelength
  – A very strong function of \( R \)
  – A linear function of everything else

Example  Radar Can Perform Search at 1000 km Range
How Might It Be Modified to Work at 2000 km ?

Solutions
Increasing \( R \) by 3 dB (x 2) Can Be Achieved by:

1. Increasing \( P_{av} \) by 12 dB (x 16)
   or 2. Increasing Diameter by 6 dB (\( A_e \) by 12 dB)
   or 3. Increasing \( t_s \) by 12 dB
   or 4. Decreasing \( \Omega \) by 12 dB
   or 5. Increasing \( \sigma \) by 12 dB
   or 6. An Appropriate Combination of the Above
Search Radar Performance

![Diagram showing the performance of different search radars based on average power, (equivalent) antenna diameter, and range.](image)

- **ASR-9**
- **ARSR-4**
- **WSR-88D/NEXRAD**
- **ASDE-3**
- **TDWR**

**Key Points**:
- **Search 1 sr in 10 sec for 1 sq m Target**: $S/N = 15$ dB
- **Loss = 10 dB**
- **T = 500 deg**

**Legend**:
- **R = 1 km**
- **R = 10 km**
- **R = 30 km**
- **R = 100 km**
- **R = 300 km**
- **R = 1000 km**
- **R = 3000 km**

*Courtesy of MIT Lincoln Laboratory. Used with permission.*
Search Radar Performance

- **Average Power (W)**
  - 100 K
  - 10 K
  - 1 K
  - 100
  - 10
  - 1

- **(Equivalent) Antenna Diameter (m)**
  - R = 10 km
  - R = 30 km
  - R = 100 km
  - R = 300 km
  - R = 1000 km
  - R = 3000 km

- **Radar Systems**
  - ASR-4
  - ASR-9
  - WSR-88D/NEXRAD
  - TDWR
  - ASDE-3

- **Search 1 sr In 10 sec for 1 sq m Target**
  - S/N = 15 dB
  - Loss = 10 dB
  - T = 500 deg

- **Airport Surface Detection Equipment**
  - ASDE-3

- **Courtesy of MIT Lincoln Laboratory**
  Used with Permission

- **Courtesy Target Corporation**
Search Radar Performance

- **ARSR-4**
  - Air Route Surveillance Radar

- **WSR-88D/NEXRAD**

- **ASR-9**
  - TDWR

- **ASDE-3**

- **R = 100 km**
  - **ARSR-4**

- **R = 30 km**
  - **ASDE-3**

- **R = 10 km**
  - **ASR-9**

- **R = 300 km**

- **R = 1000 km**

- **R = 3000 km**

**Search 1 sr In 10 sec for 1 sq m Target**
- **S/N = 15 dB**
- **Loss = 10 dB**
- **T = 500 deg**

**ARSR-4 Antenna**
- (without Radome)

**Courtesy of Northrop Grumman. Used with Permission.**

<table>
<thead>
<tr>
<th>Average Power (W)</th>
<th>Equivalent Antenna Diameter (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 K</td>
<td>100</td>
</tr>
<tr>
<td>10 K</td>
<td>10</td>
</tr>
<tr>
<td>1 K</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>0.1</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>0.1</td>
<td>100</td>
</tr>
</tbody>
</table>

**Search Radar Performance**

- **R = 10 km**
  - **ASDE-3**

- **R = 30 km**
  - **ASR-9**

- **R = 100 km**
  - **ARSR-4**

- **R = 300 km**

- **R = 1000 km**

- **R = 3000 km**

**Search 1 sr In 10 sec for 1 sq m Target**
- **S/N = 15 dB**
- **Loss = 10 dB**
- **T = 500 deg**

**ARSR-4 Antenna**
- (without Radome)

**Courtesy of Northrop Grumman. Used with Permission.**

**Equivalent Antenna Diameter (m)**
- 10.1 m
- 10 m
- 1 m
- 100 m
- 1000 m
- 3000 m

- **Search Radar Performance**

- **ASR-9**

- **TDWR**

- **ARSR-4**

- **WSR-88D/NEXRAD**

**Search 1 sr In 10 sec for 1 sq m Target**
- **S/N = 15 dB**
- **Loss = 10 dB**
- **T = 500 deg**

**ARSR-4 Antenna**
- (without Radome)

**Courtesy of Northrop Grumman. Used with Permission.**
Search Radar Performance

![Search Radar Performance Diagram](image)

- **ARSR-4**
- **ASR-9**
- **TDWR**
- **ASDE-3**
- **WSR-88D/NEXRAD**

**Search 1 sr In 10 sec for 1 sq m Target**
- **S/N = 15 dB**
- **Loss = 10 dB**
- **T = 500 deg**

**R = 10 km**
- **R = 30 km**
- **R = 100 km**
- **R = 300 km**
- **R = 1000 km**
- **R = 3000 km**

**Average Power (W)**
- **100 K**
- **10 K**
- **1 K**
- **100**
- **10**
- **1**

**Equivalent Antenna Diameter (m)**
- **100**
- **10**
- **1**

**Search Radar Performance Chart**

**WSR-88D / NEXRAD**

- Courtesy of NOAA.
- Courtesy of MIT Lincoln Laboratory
- Used with Permission
Search Radar Performance

(Equivalent) Antenna Diameter (m)

Average Power (W)

R = 10 km
R = 30 km
R = 100 km
R = 300 km
R = 1000 km
R = 3000 km

Search 1 sr
In 10 sec for
1 sq m Target
S/N = 15 dB
Loss = 10 dB
T = 500 deg

ASR- 9
ARSR- 4
TDWR
WSR-88D/NEXRAD
ASDE- 3

TDWR
Terminal Doppler Weather Radar

Courtesy of Raytheon.

Courtesy of MIT Lincoln Laboratory
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Outline

- Introduction
- Introduction to Radar Equation
- Surveillance Form of Radar Equation
- Radar Equation for Rain Clutter
- Radar Losses
- Example
- Summary
Radar Equation for Rain Clutter (and other Volume Distributed Targets)

- Standard radar equation
  \[ S = \frac{P_t G^2 \lambda^2 \sigma}{N (4\pi)^3 R^4 k T_s B_n L} \]

- If the target is a diffuse scatterer (e.g. rain), which completely fills the range-azimuth-elevation cell of the radar, then the radar cross section of the target takes the form:
  \[ \sigma = \eta V \quad \text{and} \quad V = \frac{\pi}{4} (R \theta_B) (R \phi_B) \left( \frac{c \tau}{2} \right) \frac{1}{2 \ln_e 2} \]

- And the radar equation becomes:
  \[ S = \frac{P_t G \lambda^2 c \tau \eta}{1024 (\ln_e 2) R^2 k T_s B_n L} \]

- Note, for Gaussian antenna pattern
  \[ G \approx \frac{\pi^2}{\theta_B \phi_B} \]

- Note, that volume distributed backscatter is a function of \(1/R^2\) rather than the usual \(1/R^4\)
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System Loss Terms in the Radar Equation

Transmit Losses
- Radome
- Circulator
- Waveguide Feed
- Waveguide
- Antenna Efficiency
- Beam Shape
- Low Pass Filters
- Rotary Joints
- Scanning
- Atmospheric
- Quantization
- Field Degradation

Receive Losses
- Radome
- Circulator
- Waveguide Feed
- Waveguide
- Combiner
- Receiver Protector
- Rotary Joints
- Transmit / Receive Switch
- Antenna Efficiency
- Beam Shape
- Scanning
- Doppler Straddling
- Range Straddling
- Weighting
- Non-Ideal Filter
- CFAR
- Quantization
- Atmospheric
- Field Degradation
Major Loss Terms in Radar Equation

• Beam Shape Loss
  – Radar return from target with scanning radar is modulated by shape of antenna beam as it scans across target. Can be 2 to 4 dB

• Scanning Antenna Loss
  – For phased array antenna, gain of beam less than that on boresite

• Inputs to System Noise Temperature
  – Noise received by antenna
    Local RF noise
    Galactic noise
  – Receiver noise factor
  – Receive waveguide losses
  – Antenna ohmic losses
Nature of Beam Shape Loss

Radar Equation assumes n pulses are integrated, all with gain G.

Except for the pulse at the center of the beam, the actual pulses illuminate the target with a gain less than the maximum.

(Adapted from Skolnik, Reference 1, p 82)
Major Loss Terms in Radar Equation

• Waveguide and Microwave Losses
  – Transmit waveguide losses (including feed, etc)
  – Rotary joints, circulator, duplexer

• Signal Processing Loss
  – Range and Doppler Weighting
  – A /D Quantization Losses
  – Adaptive thresholding (CFAR) Loss
  – Range straddling Loss

• Lens Effect Loss
  – Refraction in atmosphere causes spreading of beam and thus degradation in S/N

• Atmospheric Attenuation Loss
  – Attenuation as radar beam travels through atmosphere (2 way loss)
## Rectangular Waveguide Attenuation

<table>
<thead>
<tr>
<th>Frequency Band</th>
<th>Frequency Range of Dominant TE(_{10}) Mode (GHz)</th>
<th>Attenuation - Lowest to Highest Frequency (dB/100 ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UHF</td>
<td>0.35 - 0.53</td>
<td>0.054 - 0.034</td>
</tr>
<tr>
<td>L Band</td>
<td>0.96 - 1.44</td>
<td>0.20 - 0.135</td>
</tr>
<tr>
<td>S Band</td>
<td>2.6 - 3.95</td>
<td>1.10 - 0.75</td>
</tr>
<tr>
<td>C Band</td>
<td>3.95 - 5.85</td>
<td>2.07 - 1.44</td>
</tr>
<tr>
<td>X Band</td>
<td>8.2 - 12.40</td>
<td>6.42 - 4.45</td>
</tr>
<tr>
<td>K(_u) Band</td>
<td>12.4 - 18.0</td>
<td>9.58 - 8.04</td>
</tr>
<tr>
<td>K(_a) Band</td>
<td>26.5 - 40.0</td>
<td>21.9 - 15.0</td>
</tr>
</tbody>
</table>

(Adapted from Volakis, Reference 7, pp 51-40 to 51-41)
Lens Loss vs. Range

- The gradient of atmospheric refraction at lower elevation angles, causes a spreading of the radar beam, and thus a small diminishment radar power
- This lens loss is frequency independent
- It is significant only for targets that are at long range.

(Adapted from Blake, Reference 5, p 192)
Loss Due to Atmospheric Attenuation

Attenuation vs. Frequency

- Two way Attenuation through Entire Troposphere (dB)
- Radar Frequency (MHz)

Attenuation vs. Range to Target (X-Band 10 GHz)

- Two way Attenuation to Target (dB)
- Radar to Target Distance (nmi.)

0, 1, 5, 30 deg

(Adapted from Blake, see Reference 5, p 192)
Major Loss Terms in Radar Equation

- **Bandwidth Correction Factor**
  - Receiver not exact matched filter for transmitted pulse

- **Integration Loss**
  - Non coherent integration of pulses not as efficient as coherent integration

- **Fluctuation Loss**
  - Target return fluctuates as aspect angle changes relative to radar

- **Margin (Field Degradation) Loss**
  - Characteristics of radar deteriorates over time (~3 dB not unreasonable to expect over time)
    - Water in transmission lines
    - Weak or poorly tuned transmitter tubes
    - Deterioration in receiver noise figure
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• Radar Losses

• Examples

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Radar Equation - Examples

- Airport Surveillance Radar
  - 0 th order
  - Back of the envelope

- Range Instrumentation Radar
  - A more detailed calculation
Example - Airport Surveillance Radar

- Problem: Show that a radar with the parameters listed below, will get a reasonable S/N on an small aircraft at 60 nmi.

Radar Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>60 nmi</td>
</tr>
<tr>
<td>Aircraft cross section</td>
<td>1 m²</td>
</tr>
<tr>
<td>Peak Power</td>
<td>1.4 Megawatts</td>
</tr>
<tr>
<td>Duty Cycle</td>
<td>0.000525</td>
</tr>
<tr>
<td>Pulsewidth</td>
<td>0.6 microseconds</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>1.67 MHz</td>
</tr>
<tr>
<td>Frequency</td>
<td>2800 MHz</td>
</tr>
<tr>
<td>Antenna Rotation Rate</td>
<td>12.7 RPM</td>
</tr>
<tr>
<td>Pulse Repetition Rate</td>
<td>1200 Hz</td>
</tr>
<tr>
<td>Antenna Size</td>
<td>4.9 m wide by 2.7 m high</td>
</tr>
<tr>
<td>Azimuth Beamwidth</td>
<td>1.35 °</td>
</tr>
<tr>
<td>System Noise Temp.</td>
<td>950 ° K</td>
</tr>
</tbody>
</table>

\[ \lambda = \frac{c}{f} = 0.103 \text{ m} \]

\[ G = \frac{4\pi A}{\lambda^2} = 15670 \]

\[ = 42 \text{ dB, (actually 33 dB with beam shaping losses)} \]

Number of pulses per beamwidth = 21

Assume Losses = 8 dB

Courtesy of MIT Lincoln Laboratory
Used with Permission
Example - Airport Surveillance Radar

\[
\frac{S}{N} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_s B_n L}
\]

\begin{align*}
P_t &= 1.4 \text{ Megawatts} \\
G &= 33 \text{ dB} = 2000 \\
\lambda &= .1 \text{ m} \\
\sigma &= 1 \text{ m}^2 \\
k &= 1.38 \times 10^{-23} \text{ w} / \text{ Hz} \circ \text{ K} \\
T_s &= 950 \circ \text{ K} \\
B_n &= 1.67 \text{ MHz} \\
L &= 8 \text{ dB} = 6.3 \\
R &= 111,000 \text{ m}
\end{align*}

\[
\frac{(1.4 \times 10^6 \text{ w })(2000)(2000)(.1\text{m})(.1\text{m})(1\text{m}^2)}{(1984)(1.11 \times 10^5 \text{ m})^4 (1.38 \times 10^{-23} \text{ w} / \text{ Hz} \circ \text{ K})(950 \circ \text{ K})(6.3)(1.67 \times 10^6 \text{ Hz})}
\]

\[
\frac{5.6 \times 10^{+6+3+3-1-1}}{415 \times 10^{+3+20-23+2+6}} = \frac{5.6 \times 10^{+10}}{4.15 \times 10^{+2+3+20-23+2+6}} = \frac{5.6 \times 10^{+10}}{4.15 \times 10^{+10}} = 1.35 = 1.3 \text{ dB}
\]

\[
S / N = 1.3 \text{ dB per pulse (21 pulses integrated)} \Rightarrow S / N \text{ per dwell} = 14.5 \text{ dB} + 13.2 \text{ dB}
\]

Courtesy of MIT Lincoln Laboratory Used with Permission
### Example - Airport Surveillance Radar

**dB Method**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Power</td>
<td>1.4 MW</td>
<td></td>
</tr>
<tr>
<td>(Gain)²</td>
<td>33 db</td>
<td></td>
</tr>
<tr>
<td>(Wavelength )²</td>
<td>0.1 m</td>
<td></td>
</tr>
<tr>
<td>Cross section</td>
<td>1 m²</td>
<td></td>
</tr>
<tr>
<td>(4π)³</td>
<td>1984</td>
<td></td>
</tr>
<tr>
<td>(Range )⁴</td>
<td>111 km</td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>1.38 x 10⁻²³ w / Hz ° K</td>
<td></td>
</tr>
<tr>
<td>System Temp</td>
<td>950</td>
<td></td>
</tr>
<tr>
<td>Losses</td>
<td>8 dB</td>
<td></td>
</tr>
<tr>
<td>Bandwidth</td>
<td>1.67 MHz</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 356.1</td>
<td>- 354.8</td>
</tr>
<tr>
<td></td>
<td>+ 1.3 dB</td>
<td></td>
</tr>
</tbody>
</table>

\[
S / N = 1.3 \text{ dB per pulse (21 pulses integrated)} \Rightarrow S / N \text{ per dwell} = 14.5 \text{ dB ( + 13.2 dB)}
\]

Courtesy of MIT Lincoln Laboratory
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Example # 2 – Range Instrumentation Radar

• Problem: For a C-band pulsed radar with a 6.5 m dish antenna and 1,000 kW of peak power (0.1% duty cycle), what is the maximum detection range on a target with 0 dBsm cross section, a Pd of .9, and Pfa of 10^-6 (Assume a Swerling Case 1 target fluctuations and a 1 µsec pulse) ?

**Radar Parameters**

- Maximum Detection Range: ?? km
- Probability of Detection: .9
- Probability of False Alarm: 10^-6
- Target Cross Section: 0 dBsm (1 m²)
- Target Fluctuations: Swerling Case 1
- Peak Power: 1,000 Kilowatts
- Duty Cycle: 0.1%
- Pulsewidth: 1 microsecond
- Frequency: 5,500 MHz
- Antenna Size: 6.5 m dish
- Number of Pulses Integrated: 50
Detection Statistics for Swerling Case 1
(Probability of Detection = 0.9)

For Coherent Integration

\[ \frac{S}{N}_{\text{TOTAL}} = \frac{S}{N}_{\text{PER PULSE}} \]

\[ \frac{S}{N}_{\text{PER PULSE}} = 21.2 - 17.0 = 4.2 \, \text{dB} \]

\[ \frac{S}{N} = 21.2 \, \text{dB} \]

\[ P_{fa} = 10^{-6} \]

(Adapted from Blake in Skolnik, see Reference 4, p 192)
## Radar Equation Example #2

### Radar and Target Parameters – Inputs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Natural Units</th>
<th>(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Power (kilowatts)</td>
<td>1,000</td>
<td>60.0</td>
</tr>
<tr>
<td>Pulse Duration (microseconds)</td>
<td>1.0</td>
<td>-60.0</td>
</tr>
<tr>
<td>Noise Bandwidth (MHz)</td>
<td>1.0</td>
<td>60.0</td>
</tr>
<tr>
<td>Transmit Antenna Gain (dB)</td>
<td></td>
<td>49.6</td>
</tr>
<tr>
<td>Receive Antenna Gain (dB)</td>
<td></td>
<td>49.6</td>
</tr>
<tr>
<td>Frequency (GHz)</td>
<td>5.5</td>
<td></td>
</tr>
<tr>
<td>Wavelength (meters)</td>
<td>5.45</td>
<td>-25.3</td>
</tr>
<tr>
<td>Single Pulse Signal to Noise Ratio</td>
<td></td>
<td>4.2</td>
</tr>
<tr>
<td>Target Radar Cross Section (meters$^2$)</td>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$k$ - Boltzmann’s Constant $1.38 \times 10^{-23}$ (w / Hz °K)</td>
<td></td>
<td>-228.6</td>
</tr>
<tr>
<td>$(4\pi)^3$</td>
<td></td>
<td>33.0</td>
</tr>
<tr>
<td>System Noise Temperature (°K)</td>
<td>598.2</td>
<td>27.8</td>
</tr>
<tr>
<td>Total System Losses</td>
<td></td>
<td>9.0</td>
</tr>
<tr>
<td>Range (kilometers)</td>
<td>519</td>
<td></td>
</tr>
</tbody>
</table>

### Antenna

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency</td>
<td>65 %</td>
</tr>
<tr>
<td>Diameter (meters)</td>
<td>6</td>
</tr>
<tr>
<td>Gain (dB)</td>
<td>49.6</td>
</tr>
</tbody>
</table>
## Radar Equation # 2  System Losses

<table>
<thead>
<tr>
<th>System Losses</th>
<th>dB</th>
<th>Transmit Losses</th>
<th>dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth Correction Factor (dB)</td>
<td>0.70</td>
<td>Circulator (dB)</td>
<td>0.40</td>
</tr>
<tr>
<td>Transmit Loss (dB)</td>
<td>1.30</td>
<td>Switches (dB)</td>
<td>0.40</td>
</tr>
<tr>
<td>Scanning Antenna Pattern Loss (dB)</td>
<td>0.00</td>
<td>Transmission Line</td>
<td>0.50</td>
</tr>
<tr>
<td>Signal Processing Losses (dB)</td>
<td>1.90</td>
<td></td>
<td>1.30</td>
</tr>
<tr>
<td>Atmospheric Attenuation Loss (dB)</td>
<td>1.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lens Effect Loss (dB)</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Integration Loss (dB)</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Target Fluctuation Loss (dB)</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Margin / Field Degradation Loss (dB)</td>
<td>3.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Loss Budget (dB)</td>
<td>8.95</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

## Signal Processing Losses (dB)

<table>
<thead>
<tr>
<th>Losses</th>
<th>dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold Loss (dB)</td>
<td>0.50</td>
</tr>
<tr>
<td>A/D Quantization Loss (dB)</td>
<td>0.10</td>
</tr>
<tr>
<td>Range Straddling Loss</td>
<td>0.20</td>
</tr>
<tr>
<td>Weighting Loss</td>
<td>1.10</td>
</tr>
</tbody>
</table>

## Loss – Input to System Noise Temperature

- Receiver Noise Factor (dB) 4.00
- Antenna Ohmic Loss (dB) 0.20
- Receive Transmission Line loss (dB) 0.40
- Sky Temperature (°K) 50.00

C-Band at 3°

\[
T_s = T_a + T_r + L_r T_e = 598.2°K
\]

\[
T_a = (0.88 T_{sky} - 254)/(L_a + 290)
\]

\[
T_r = T_{tr} (L_r - 1) \quad \text{and} \quad T_e = T_o (F_n - 1)
\]
Outline

- Introduction
- Introduction to Radar Equation
- Surveillance Form of Radar Equation
- Radar Equation for Rain Clutter
- Radar Losses
- Examples
- Summary
Cautions in Using the Radar Equation (1)

• The radar equation is simple enough, that just about anyone can learn to use and understand

• Unfortunately, the radar equation is complicated enough that anyone can mess it up, particularly if you are not careful
  – See next viewgraph for relevant advice
Cautions in Using the Radar Equation (2)

The Sanity Check

• Take a Candidate Radar Equation

• Check it Dimensionally
  – $R$ and $\lambda$ are distance
  – $\sigma$ is distance squared
  – $P_t$ is energy / time
  – $S/N$, $G$, and $L$ are dimensionless
  – $kT_s$ is energy
  – $B_n$ is (time)$^{-1}$

• Check if Dependencies Make Sense
  – Increasing Range and $S/N$ make requirements tougher
  – Increasing Power and Antenna Gain make radar more capable
  – Decreasing Wavelength and Radar Cross Section make the radar less capable

\[
\frac{S}{N} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 kT_s B_n L}
\]
Radar Equation and the Detection Process

Radar Parameters
- Transmitter Power
- Antenna Gain
- Frequency
- Pulse Width
- Waveform

Target Fluctuation Statistics
- Swerling Model 1, 2, 3, or 4
- Other

Type of Detection
- Linear
- Square Law

Detection

Radar Equation

Signal to Noise Ratio (S/N)

Detection Threshold
- Constant
- Adaptive

Probability Of Detecting Target
- \( P_D \)

Probability Of False Alarm
- \( P_{FA} \)

Range
- Radar to Target

Target Characteristics
- Cross Section vs. Angle and Frequency

Properties of Propagation Medium
- Attenuation vs. Frequency
- Rain Requirements

Noise Statistics
- Gaussian
- Other

Other
Summary

• The radar equation relates:
  – Radar performance parameters - Detection range, S/N, etc.
  and
  – Radar design parameters - Transmitter power, antenna size, etc.

• There are different forms of the radar equations for different radar functions
  – Search, Track

• Scaling of the radar equation allows the radar designer to understand how the radar parameters may change to accommodate changing requirements

• Be careful, if the radar equation leads to unexpected results
  – Do a sanity check!
    Look for hidden variables or constraints
    Compare parameters with those of a real radar
References

Contributors

- Dr Stephen D. Weiner
- Dr. Claude F. Noiseux
Homework Problems

  – Problem 1-5
  – Problem 1-6
  – Problem 2-22
  – Problem 2-24
  – Problem 2-25