Radar Systems Engineering
Lecture 14
Airborne Pulse Doppler Radar

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Guest Lecturer
Examples of Airborne Radars

- **F-16**
  - APG-66, 68
  - Courtesy of US Air Force

- **Boeing 737 AEW&C**
  - Courtesy of milintelTR

- **E-2C**
  - APS-125
  - Courtesy of US Navy

- **JOINT STARS E-8A**
  - APY-3
  - Courtesy of US Air Force

- **AWACS**
  - E-3A
  - APY-1
  - Courtesy of US Air Force
Outline

• Introduction
  – The airborne radar mission and environment
    Clutter is the main issue

• Different airborne radar missions
  – Pulse Doppler radar in small fighter / interceptor aircraft
    F-14, F-15, F-16, F-35
  – Airborne, surveillance, early warning radars
    E-2C (Hawkeye), E-3 (AWACS), E-8A (JOINT STARS)
  – Airborne synthetic aperture radar
    Military and civilian remote sensing missions
    To be covered in lecture 19, later in the course

• Summary
Block Diagram of Radar System

Clutter as seen from an airborne platform, Signal waveforms, and Doppler processing will be the focus in this lecture.

Photo Image
Courtesy of US Air Force
First Use of Airborne Radars

- US APS-3 Radar with Dish Antenna - 3 cm wavelength
- German “Lichtenstein” Radar Dipole array – 75 / 90 cm wavelength

- When they were introduced on airborne platforms during World War II, they were used to detect hostile aircraft at night in either a defensive or an offensive mode

Courtesy of US Navy

Courtesy of Department of Defense
Role of Airborne Military Radars

• Missions and Functions
  – Surveillance, Tracking, Fire Control
  – Reconnaissance
  – Intelligence

• Examples
  – Air-to-air fighter combat
    Aircraft interception (against air breathing targets)
  – Airborne Early warning
  – Air to ground missions
  – Close air support
  – Ground target detection and tracking

• Radar modes
  – Pulse Doppler radar
  – Synthetic Aperture radar
  – Displaced Phase Center Antenna (DPCA)
  – Ground Moving Target Indication
Geometry of Airborne Clutter

• Key components of the ground clutter echo from radar’s on an airborne platform:
  – Main beam of antenna illuminates the ground
  – Antenna sidelobes illuminate clutter over a wide range of viewing angles
  – Altitude return reflects from the ground directly below the radar

The Doppler frequency distributions of these effects and how they affect radar performance differ with:
  1. radar platform velocity (speed and angle), and
  2. the geometry (aspect angle of aircraft relative to ground illumination point)
Airborne Radar Clutter Spectrum

No Doppler Ambiguities  \( V_P \) and \( V_T \) in same vertical plane

- Antenna Mainlobe
- Antenna Sidelobes
- Outgoing Target

Relative Power (dB)

- Clutter Free
- Sidelobe Clutter
- Mainlobe Clutter
- Noise
- Outgoing Target

Doppler Frequency

\[ -\frac{2V_P}{\lambda} \quad 0 \quad \frac{2V_P}{\lambda} \]
Airborne Radar Clutter Spectrum

No Doppler Ambiguities

\( V_P \) and \( V_T \) in same vertical plane

\[ \frac{-2V_P}{\lambda} \quad 0 \quad \frac{2V_P}{\lambda} \]

Relative Power (dB)

- Clutter Free
- Sidelobe Clutter
- Mainlobe Clutter
- Noise

Doppler Frequency

- Incoming Target
- Noise
- Clutter Free

Antenna Mainlobe

Incoming Target

Antenna Sidelobes
Airborne Radar Clutter Characteristics

- Illustrative example
- Without Pulse-Doppler ambiguities

- Doppler frequency of mainbeam clutter depends on scan direction
- Doppler frequency of target depends on scan direction and target aspect angle

Viewgraph Courtesy of MIT Lincoln Laboratory Used with permission
Constant Range Contours on the Ground

Range to Ground Scenario

Lines of Constant Range to Ground

$R_s^2 = h^2 + R_g^2$

- The projections on the ground of the lines of constant range are a set circles
Constant Doppler Velocity Contours on the Ground

\[
V_C = V_P \cos \alpha
\]

\[
= V_P \cos \theta \sin \phi
\]

\[
V_C = \text{Clutter velocity}
\]

\[
V_P = \text{Platform velocity}
\]

- The projections on the ground of the lines of constant Doppler velocity are a set hyperbolae

\[
f_D = \frac{2V_C \cos \alpha}{\lambda}
\]
Constant Doppler Contours on Ground

- The lines of constant Doppler frequency/velocity are called “Isodops”
- The equation for the family of hyperbolae depend on:
  - Airborne radar height above ground
  - Angle between airborne radar velocity and the point on the ground that is illuminated
  - Wavelength of radar

\[
V_C = 0 \\
V_C = -V_P \\
V_C = +V_P
\]
Range-Doppler Ground Clutter Contours

- **Range Contours**
  - Circles
  - $V_C = 0$

- **Doppler Contours**
  - Hyperbolae
  - $V_C = -V_p$
  - $V_C = +V_p$

$V_C = 0$
Range-Doppler Ground Clutter Contours

Range Contours
- Circles

Doppler Contours
- Hyperbolae

Cross Range

Down Range

Range – Doppler Cell on Ground

Up Range

ΔR

Δf₀

Power

Doppler Frequency

x
Unambiguous Doppler Velocity and Range

\[ V_B = \frac{\lambda f_{\text{PRF}}}{2} \]

and

\[ R_U = \frac{c}{2 f_{\text{PRF}}} \]

Yields

\[ V_B = \frac{\lambda c}{4 R_U} \]
## Classes of Pulse Doppler Radars

<table>
<thead>
<tr>
<th>PRF</th>
<th>Range Measurement</th>
<th>Doppler Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low PRF</td>
<td><strong>Unambiguous</strong></td>
<td><strong>Highly Ambiguous</strong></td>
</tr>
<tr>
<td>Medium PRF</td>
<td>Ambiguous</td>
<td>Ambiguous</td>
</tr>
<tr>
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<td><strong>Highly Ambiguous</strong></td>
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</tbody>
</table>
Missions for Airborne Military Radars

“The Big Picture”

- **Fighter / Interceptor Radars**
  - Antenna size constraints imply frequencies at X-Band or higher
  - Reasonable angle beamwidths
  - This implies Medium or High PRF pulse Doppler modes for look down capability

- **Wide Area Surveillance and Tracking**
  - Pulse Doppler solutions
    - Low, Medium and/or High PRFs may be used depending on the specific mission
  - E-2C UHF
  - AWACS S-Band
  - Joint Stars X-Band

- **Synthetic Aperture Radars** will be discussed in a later lecture
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    F-14, F-15, F-16, F-35
    High PRF Modes
    Medium PRF Modes
  – Airborne, surveillance, early warning radars
    E-2C (Hawkeye), E-3 (AWACS), E-8A (JOINT STARS)
  – Airborne synthetic aperture radar
    Military and civilian remote sensing missions
    To be covered in lecture 19, later in the course

• Summary
Photographs of Fighter Radars

APG-65 (F-18)
 Courtesy of Raytheon
 Used with permission

APG-66 (F-16)
 Courtesy of Northrop Grumman
 Used with Permission

APG-63 V(2) (F-15C)
 Radar built by Raytheon
 Courtesy of Boeing
 Used with permission

Active Electronically Scanned Arrays (AESA)

APG-81 (F-35)
 Courtesy of Northrop Grumman
 Used with Permission

APG-81 (F-35)
Courtesy of USAF
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    Military and civilian remote sensing missions
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• Summary
## Pulse Doppler PRFs

<table>
<thead>
<tr>
<th>Frequency</th>
<th>PRF Type</th>
<th>PRF Range*</th>
<th>Duty Cycle*</th>
</tr>
</thead>
<tbody>
<tr>
<td>X- Band</td>
<td>High PRF</td>
<td>100 - 300 KHz</td>
<td>&lt; 50%</td>
</tr>
<tr>
<td>X- Band</td>
<td>Medium PRF</td>
<td>10 - 30 KHz</td>
<td>~ 5%</td>
</tr>
<tr>
<td>X- Band</td>
<td>Low PRF</td>
<td>1 - 3 KHz</td>
<td>~.5%</td>
</tr>
<tr>
<td>UHF</td>
<td>Low PRF</td>
<td>300 Hz</td>
<td>Low</td>
</tr>
</tbody>
</table>

* Typical values only; specific radars may vary inside and outside these limits
High PRF Mode

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Example: PRF = 150 KHz  Duty Cycle = 35%

PRI = 6.67 μsec  Pulsewidth = 2.33 μsec

Unambiguous Range = 1 km
Unambiguous Doppler Velocity = 4,500 knots

For high PRF mode:
- Range – Highly ambiguous
  Range ambiguities resolved using techniques discussed in Lecture 13
- Doppler velocity – Unambiguous
  For nose on encounters, detection is clutter free
- High duty cycle implies significant “Eclipsing Loss”
  Multiple PRFs, or other techniques required
High PRF Mode – Range Eclipsing

• High PRF airborne radars tend to have a High Duty cycle to get high energy on the target
  – Pulse compression used
  

Eclipsing loss is caused because the receiver cannot be receiving target echoes when the radar is transmitting
  – Can be significant for high duty cycle radars
  – Loss can easily be 1-2 dB, if not mitigated
High PRF Pulse Doppler Radar

- No Doppler velocity ambiguities, many range ambiguities
  - Significant range eclipsing loss

- Range ambiguities can be resolved by transmitting 3 redundant waveforms, each at a different PRF
  - Often only a single range gate is employed, but with a large Doppler filter bank

- The antenna side lobes must be very low to minimize sidelobe clutter
  - Short range sidelobe clutter often masks low radial velocity targets

- High closing speed aircraft are detected at long range in clutter free region

- Range accuracy and ability to resolve multiple targets can be poorer than with other waveforms
Outline

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    F-14, F-15, F-16, F-35
    High PRF Modes
    Medium PRF Modes

  – Airborne, surveillance, early warning radars
    E-2C (Hawkeye), E-3 (AWACS), E-8A (JOINT STARS)

  – Airborne synthetic aperture radar
    Military and civilian remote sensing missions
    To be covered in lecture 19, later in the course

• Summary
## Medium PRF Mode

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<th>PRF Type</th>
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<th>Duty Cycle*</th>
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<tbody>
<tr>
<td>X- Band</td>
<td>Medium PRF</td>
<td>10 - 30 KHz</td>
<td>~ 5%</td>
</tr>
</tbody>
</table>

**Example:** 7 PRF = 5.75, 6.5, 7.25, 8, 8.75, 9.5 & 10.25 KHz  
(From Figure 3.44 in text)  
Range Ambiguities = ~14 to 26 km  
Blind Speeds = ~175 to 310 knots

- For the medium PRF mode:
  - Clutter and target ambiguities in range and velocity
  - Clutter from antenna sidelobes is an significant issue
Clear Velocity Regions for a Medium PRF Radar

The multiple PRFs (typically 7) and their associated higher radar power are required to obtain sufficient detections to unravel range and velocity ambiguities in medium PRF radars.
Medium PRF Mode

• In the Doppler domain, the target and clutter alias (fold down) into the range 0 to PRF1, PRF2, etc.
  – Because of the aliasing of sidelobe clutter, medium PRF radars should have very low sidelobes to mitigate this problem

• In the range domain similar aliasing occurs
  – Sensitivity Time Control (STC) cannot be used to reduce clutter effects (noted in earlier lectures)

• Range and Doppler ambiguity resolution techniques described in previous lecture
Medium PRF Pulse Doppler Radar

• Both range and Doppler ambiguities exist
  – Seven or eight different PRFs must be used
  – Insures target seen at enough Doppler frequencies to resolve range ambiguities
  – Transmitter larger because of redundant waveforms used to resolve ambiguities

• There is no clutter free region
  – Fewer range ambiguities implies less of a problem with sidelobe clutter
  – Antenna must have low sidelobes to reduce sidelobe clutter

• Often best single waveform for airborne fighter / interceptor

• More range gates than high PRF, but fewer Doppler filters for each range gate

• Better range accuracy and Doppler resolution than high PRF systems
Outline

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    E-2C (Hawkeye), E-3 (AWACS), E-8A (JOINT STARS)

  – Airborne synthetic aperture radar
    Military and civilian remote sensing missions
    To be covered in lecture 19, later in the course

• Summary
Airborne Surveillance & Tracking
Radars

• Missions and Functions
  – Surveillance, Tracking, Fire Control
  – Reconnaissance
  – Intelligence

• Examples
  – Airborne early warning
  – Ground target detection and tracking

• Radar modes
  – Pulse Doppler radar
  – Synthetic Aperture radar
  – Displaced Phase Center Antenna (DPCA)
  – Other modes/techniques

Elevated radar platforms provide long range and over the horizon coverage of airborne and ground based targets.
Examples of Airborne Radars

- **Boeing 737 AEW&C**
  - Courtesy of milintelTR

- **Global Hawk**
  - Courtesy of US Air Force

- **JOINT STARS E-8A APY-3**
  - Courtesy of US Air Force

- **E-2C APS-125**
  - Courtesy of US Navy

- **AWACS E-3A APY-1**
  - Courtesy of US Air Force
• Elevating the radar can extend radar coverage well out over the horizon

• Range Coverage -400 km to 800 km
  – Ground based radars ~400 km
  – Airborne radar ~800 km

• Issues
  – High acquisition and operating costs
  – Limited Antenna size
  – Radar Weight and prime power
  – More challenging clutter environment
Characteristics of Ground Clutter
(from Airborne Platform)

Ground Clutter Doppler Frequency

\[ f_C = \frac{2 v_P}{\lambda} \cos \alpha = \frac{2 v_P}{\lambda} \cos \theta \sin \phi \]

Doppler Frequency Width
(Sidelobe + Main Beam Clutter)

\[ \Delta f_{SL+ML} = \frac{4 v_P}{\lambda} \]

Doppler Frequency Width of Main Beam Clutter (Null to Null)

\[ \Delta f_{MB} = \frac{4 v_P}{\lambda} \frac{\lambda}{L} = \frac{4 v_P}{L} \]
Spread of Main Beam Clutter

• Doppler frequency of clutter return depends on angle of clutter with velocity vector of aircraft

• Doppler frequency of clutter return at center of beam

\[ f_C = \frac{2 V_P}{\lambda} \cos \theta \]

• Doppler spread of main beam clutter can be found by differentiating this equation

\[ \Delta f_C = \frac{2 V_P}{\lambda} \theta_B \sin \theta \]

Spread of Main Beam Clutter Maximum at \( \theta = 90^\circ \)

Depression angle of beam neglected

Adapted from Skolnik Reference 1
Clutter Spread with a UHF Airborne Radar

- Both the width of the clutter spectra and its center frequency depend on the angle $\theta$.

- When the antenna points in the direction of the platform velocity vector, the Doppler shift of the clutter echo is maximum, but the width of the spectrum is theoretically zero.

- When the antenna is directed in the direction perpendicular to the direction of the platform velocity, the clutter center frequency is zero, but the spread is maximum.

Adapted from Skolnik Reference 1
• PRF = 360 Hz corresponds to a maximum unambiguous range of 225 nmi
• A relatively large portion of the frequency domain (Doppler space) is occupied by the clutter spectrum because of platform motion
• The widening of the clutter needs to be reduced in order for standard clutter suppression techniques to be effective
• There are 2 effects that can seriously degrade the performance of a radar on a moving platform
  – A non-zero Doppler clutter shift
  – A widening of the clutter spectrum

• These may be compensated for by two different techniques
  – TACCAR (Time Averaged Clutter Coherent Airborne Radar)
    The change in center frequency of the clutter spectrum
  – DPCA (Displaced Phase Center Antenna)
    The widening of the clutter spectrum

• Radars which have used these techniques, over the years, to compensate for platform motion are Airborne Early Warning radars
Compensation for Clutter Doppler Shift

- **TACCAR (Time Averaged Clutter Coherent Airborne Radar)**
  - Also called “Clutter Lock MTI”

- The Doppler frequency shift from ground clutter can be compensated by using the clutter echo signal itself to set the frequency of the reference oscillator (or coho)
  - This process centers the ground clutter to zero Doppler frequency
  - The standard MTI filter (notch at zero Doppler) attenuates the ground clutter

- This technique has been used in ground based radars to mitigate the effect of moving clutter
  - Not used after the advent of Doppler filter processing
AEW Advances - E-2D and MP-RTIP

- **E-2D**
  - Mechanically Rotating Active Electronically Scanned Antenna (AESA)
  - Space Time Adaptive Processing (STAP)

- **MP-RTIP**
  - “Multi-Platform Radar Technology Insertion Program”
  - Originally Joint Stars Upgrade Program
    - Global Hawk and then a wide area surveillance aircraft
  - Advanced ground target surveillance capability
E-3A Sentry - AWACS

• AWACS Radar (S-Band)
  – Mission – Long range Surveillance, Command and Control for air tactical environment
  – Radar System Improvement Program (RSIP)
    Advanced pulse Doppler waveforms
    Pulse compression added
    Detection range doubled (over original radar)

Radar APY-2

S-Band (10 cm wavelength)

Range >250 miles

High PRF waveform to reject clutter in look down mode

Long range beyond the horizon surveillance mode

Maritime surveillance mode

See reference 1
AWACS (APY-1/2) Antenna

- Phased array – 26 ft by 4.5 ft ultralow sidelobe array
  Elliptically shaped
- 28 slotted waveguides with a total of over 4000 slots
- Antenna is mechanically scanned 360° in azimuth
- Uses 28 ferrite reciprocal phase shifters to scan in elevation
- 10 sec rotation (data) rate

See Skolnik reference 1
If the aircraft motion is exactly compensated by the movement of the phase center of the antenna beam, then there will be no clutter spread due to aircraft motion, and the clutter can be cancelled with a two pulse canceller.
A mechanically rotating antenna on a moving platform that generates two overlapping (squinted) beams can act as a DCPA when the outputs of the two squinted beams are properly combined.

- The sum and difference of the two squinted beams are taken
  - The sum is used for transmit
  - The sum and difference are used on receive

- A phase advance is added to the first pulse and a phase lag is added to the second pulse beams are taken

- The added (or subtracted) phase shift depends on aircraft velocity, the PRF, and the scan angle of the radar relative to the aircraft direction

- The two signals are then subtracted, resulting in the cancellation of the Doppler spread of the clutter
DPCA – The Math- Abbreviated

Individual Clutter Scatterer
Angle $\alpha$ off Beam 1 beam center

Beam 2

Phasor representation of clutter echoes from 2 successive pulses

$e_2 = -jE_2 \tan \eta$

Corrections applied to pulses allowing cancellation

$e_1 = jE_1 \tan \eta$

$\sum_R = \text{Sum (2 pulses) of receive signal}$

$\Delta_R = \text{Difference (2 pulses) of receive signal}$

The sum and difference of the two squinted beams are taken

The sum is used for transmit
The sum and difference are used on receive

After MUCH manipulation, the corrected received pulses become:

Pulse 1

$\sum_R (\alpha) + jk(v \sin \theta) \Delta_R (\alpha)$

Pulse 2

$\sum_R (\alpha) - jk(v \sin \theta) \Delta_R (\alpha)$

Constant $k$ accounts for differences in $\sum$ and $\Delta$ patterns, as well as a factor $4T_p/D$

For more detail see Skolnik, Reference 1, pp 166-168
Multiple Antenna Surveillance Radar (MASR)

DPCA Off

DPCA On

Viewgraph Courtesy of MIT Lincoln Laboratory
Used with permission
Joint Surveillance Target Attack Radar System (Joint STARS)

- Employs Interferometric SAR for airborne detection of ground vehicles and imaging of ground and surface targets
  - Employs APY-3, X Band radar
- Mission in wide area surveillance mode:
  - Coverage ~50,000 km²
  - Detect, locate, identify, classify, and track trucks, tanks, and other vehicles
    Can differentiate tracked and wheeled vehicles
    Can see vehicles at ranges >200 km, moving at walking speeds

Courtesy of US Air Force
Joint Stars Radar

- Radar employs a slotted array antenna 24 ft by 2 ft
  - 456 x 28 horizontally polarized elements
  - Beam scans \( \pm 60^\circ \) in azimuth; mechanically rotated in elevation
- Aperture can be used as a whole for SAR mapping
- When total aperture is divided into 3 independent apertures in the interferometric mode, it is used for moving target detection
  - Moving targets are separated from clutter by different time of arrivals of target and clutter in the 3 apertures
  - DPCA techniques are used to cancel main beam clutter
Joint Stars Moving Target Detections

Operation Desert Storm
(Feb 1991)

Courtesy of Northrop Grumman
Used with Permission
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  – Airborne synthetic aperture radar
    SAR basics to be covered in lecture 19
    Military and civilian remote sensing missions
    To be covered in lecture 19, later in the course

• Summary
Detection of Ground Moving Targets

- Ground Moving Target Indication (GMTI)
  - Low or medium PRF pulse Doppler radar used
  - PRF chosen so that Doppler region of interest is unambiguous in range and Doppler
  - $K_u$ (16 GHz) or $K_\alpha$ (35 GHz) Band often used, since fixed minimum detectable Doppler frequency will allow detection of lower velocities than X band
  - APG-67 (X-Band) in F-20 fighter has GMTI mode using medium PRF
  - AWACS has low PRF ship detection mode

- Side-Looking Airborne Radar (SLAR)
  - Standard airborne radar subtracts sequential conventional images of terrain (Non-coherent MTI) to detect moving targets
Detection of Ground Moving Targets

• Synthetic Aperture Radar (SAR) with MTI
  – SARs (discussed in lecture 19) produce excellent images of fixed targets on the ground
    Good cross range resolution obtain by processing sequential target echoes as aircraft moves a significant distance L
    Cross range resolution inversely proportional to L not antenna size D
  – Moving targets distorted and smeared in SAR image
  – Can be detected if target Doppler is greater than bandwidth of clutter echo
  – Requires high PRF to avoid aliasing issues

• Joint Stars
  – Uses interferometer for clutter suppression processing
Summary

• Difficult ground clutter environment is chief radar design driver for airborne radars
  – Elevated radar platform implies ground clutter at long range
  – Both Doppler frequency of clutter and its spread depend on radar platform motion and scan angle

• Clutter challenges with Airborne radars
  – Antenna aperture size often limits frequencies, so that ambiguous range and Doppler velocity issues arise
    Low, Medium and High PRF Modes each have unique clutter issues
  – Doppler spreading of ground clutter, particularly at broadside, viewing can degrade performance

• Sophisticated clutter suppression techniques can alleviate some of these issues
  – DPCA techniques
  – Medium and High PRF modes often imply higher power

• Active Electronically Scanned arrays and advanced signal processing techniques (STAP) offer significant new capabilities for airborne radars
Homework Problems

- From Skolnik (Reference 1)
  - Problems 3-19, 3-20, 3-21, 3-22, 3-23, and 3-24
  - Show that the maximum Doppler frequency of ground clutter as seen by an airborne radar is
    \[
    f_D \leq \frac{2V}{\lambda} \left(1 - \frac{h^2}{R^2}\right)
    \]
    Where:
    \[
    V = \text{velocity of airborne radar}
    \]
    \[
    \lambda = \text{radar wavelength}
    \]
    \[
    h = \text{height of radar above ground}
    \]
    \[
    R = \text{slant range}
    \]
  - Show that, for an airborne radar flying at a constant height above the ground, the lines of constant clutter velocity are a set of hyperbolae

The last problem is from Roger Sullivan’s previously referenced text.
References

Acknowledgements

• Niall J. Duffy
• Dr. Allen Hearn
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