Radar Systems Engineering
Lecture 7 Part 2
Radar Cross Section

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IEEE New Hampshire Section
Guest Lecturer
# Methods of Radar Cross Section Calculation

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Electromagnetic Scattering

- Two step process to determine scattered fields
  - Determine induced surface currents
  - Calculate field radiated by currents

Incident Plane Wave

Induced Surface Currents

Scattered Field
(Radiated by Induced Currents)

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Method of Moments (MoM) Overview

• The Method of Moments calculations predict the exact solution for the target RCS

• Method – Solve integral form of Maxwell’s Equations
  – Generate a surface patch model for the target
  – Transform the integral equation form of Maxwell’s equations into a set of homogeneous linear equations
  – The solution gives the surface current densities on the target
  – The scattered electric field can then be calculated in a straightforward manner from these current densities
  – Knowledge of the scattered electric field then allows one to readily calculate the radar cross section

• Significant limitations of this method
  – Inversion of the matrix to solve the homogeneous linear equations
  – Matrix size can be very large at high frequencies
    Patch size typically ~\(\lambda/10\)
Standard Spherical Coordinate System

\[ \theta, \phi, r \]

\[ (x, y, z) \]
Spherical Coordinate System for MOM Calculations

- Source currents distributed over surface $S'$
- Field observation point located at $(x, y, z)$
- Point on surface $S'$ is $(x', y', z')$

\[ \vec{R} = \vec{r} - \vec{r}' \]
Method of Moments

- Maxwell’s Equations transform to the Stratton and Chu Equations using the vector Green’s Theorem and yield:

\[
\bar{E}_S = \oint_{S'} \left[ + i \omega \mu \left( \hat{n} \times \bar{H} \right) \psi + \left( \hat{n} \times \bar{E} \right) \times \nabla \psi + \left( \hat{n} \cdot \bar{E} \right) \nabla \psi \right] dS'
\]

\[
\bar{H}_S = \oint_{S'} \left[ + i \omega \varepsilon \left( \hat{n} \times \bar{E} \right) \psi - \left( \hat{n} \times \bar{H} \right) \times \nabla \psi - \left( \hat{n} \cdot \bar{H} \right) \nabla \psi \right] dS'
\]

\[
\psi = \left[ \frac{e^{ikR}}{4\pi R} \right] = \text{Free Space Green’s Function}
\]

\[
R = |r - r'|
\]

- Free space Green’s function is an spherical wave falling off as: \( 1/R \)

- Also, note:

\[
\bar{E} = \bar{E}_1 + \bar{E}_S
\]

\[
\bar{H} = \bar{H}_1 + \bar{H}_S
\]
Method of Moments (continued once)

• On the surface of the perfectly conducting target these equations become:
  – Total tangential electric field zero at surface
  – No magnetic sources of currents or charges as source of scattered fields

• Electric Field Integral Equation (EFIE)

\[
\bar{E}_S = \oint_{S'} \left[ + \im \omega \mu (\hat{n} \times \bar{H}) \psi + (\hat{n} \cdot \bar{E}) \nabla \psi \right] dS' = \oint_{S'} \left[ + \im \omega \mu \mathbf{J} \psi + \frac{1}{\varepsilon} \rho \nabla \psi \right] dS'
\]

• Magnetic Field Integral Equation (MFIE)

\[
\bar{H}_S = \oint_{S'} (\hat{n} \times \bar{H}) \times \nabla \psi dS' = \oint_{S'} \bar{\mathbf{J}} \times \nabla \psi dS'
\]

• Causes of scattered fields
  – Scattered electric field – electric currents and charges
  – Scattered magnetic field – electric currents
Method of Moments (continued twice)

• Applying the boundary conditions for Maxwell’s Equations and the Continuity Equation to free space yields:

\[ \hat{n} \times \vec{E}_1 = -\hat{n} \times \vec{E}_S = \hat{n} \times \iint_{S'} \left[ +i \omega \mu \vec{J} \psi + \frac{+i}{\omega \varepsilon} \nabla \cdot \vec{J} \nabla \psi \right] dS' \]

\[ \hat{n} \times \vec{H}_1 = \frac{\vec{J}}{2} - \hat{n} \times \iint_{S'} \vec{J} \times \nabla \psi \, dS' \]

• Procedure to calculate the scattered electric field:
  – Convert the integral equation into a set of algebraic equations
  – Solve for induced current density using matrix algebra
  – With the current density known, the calculation of the scattered electric field, \( \vec{E}^S \), is reasonably straightforward and the cross section can be calculated:

\[ \sigma = 4 \pi R^2 \frac{|E^S|^2}{|E^1|^2} \]
Method of Moments (continued again)

• Break up the target into a set of $N$ discrete patches
  – 7 to 10 patches per wavelength

• Expand the surface current density as a set of known basis functions
  \[ \bar{J}(\bar{r}) = \sum_{n=1}^{N} I_n \bar{B}_n(\bar{r}) \]

• Define the “Magnetic Field Operator”, $L_H(\bar{J})$, as
  \[ L_H(\bar{J}) \equiv \frac{\bar{J}}{2} - \hat{n} \times \int_S \bar{J} \times \nabla \psi \, dS' \]

• Insert the series expansion of currents and bringing the sum out of the operator, we get:
  \[ L_H(\bar{J}) = \sum_{n=1}^{N} I_n \, L_H(\bar{B}_n(\bar{r})) = \hat{n} \times \bar{H}^I \]
Method of Moments (one last time)

- Multiply by the weighting vector, $\tilde{W}_m$, and integrating over the surface:

$$\oint_S \left[ \tilde{W}(\vec{r}) \cdot (\hat{n} \times \vec{H}^I) \right] dS - \sum_{n=1}^{N} I_n i \omega \mu \oint_{S'} \oint_S \tilde{W}_m \cdot L(B_n(\vec{r})) dS' dS = 0$$

  - Point Testing $\tilde{W}_m = \delta(\vec{r} - \vec{r}_m)$

  - Galerkin’s Method $\tilde{W}_m = \tilde{B}_m(\vec{r})$

- This is a set of $N$ equations in $N$ unknowns (current coefficients, $I_m$) of the form:

$$\tilde{Z} \tilde{I} = \tilde{V} \quad \rightarrow \quad \tilde{I} = \tilde{Z}^{-1} \tilde{V}$$

- The only difficulty is inversion of a very large matrix
Monostatic RCS of a Square Plate

- 15 cm x 15 cm Plate
- 6.0 GHz
- HH Polarization

Aspect Angle (degrees)

Radar Cross Section (dBsm)

Measurement
Monostatic RCS of a Square Plate

- 15 cm x 15 cm Plate
- 6.0 GHz
- HH Polarization

Aspect Angle (degrees)
-90 -60 -30 0 30 60 90

Radar Cross Section (dBsm)
-30 -20 -10 0 10 20

Measurement
Method of Moments

Patch Size
λ/4
N = 12
Surface Patch Model of JGAM for Method of Moments RCS Calculation

- 1.0 GHz  1350 unknowns

Top View

Side View

Photo of JGAM on Pylon

Courtesy of MIT Lincoln Laboratory
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Summary - Method of Moments

• Method of moments solution is exact
  – Patch size must be small enough
  – 7 to 10 samples per wavelength

• Well suited for small targets at long wavelengths
  – Example - Artillery shell at L-Band (23 cm)

• Aircraft size targets result in extremely large matrices to be inverted
  – JGAM (~ 5m length)
    1350 unknowns at 1.0 GHz
  – Typical Fighter aircraft (~ 5m length)
    A very difficult computation problem at S-Band (10 cm wavelength)
Comparison of MoM and FD-TD Techniques

- For Single Frequency RCS Predictions (perfect conductors)
  - **2-Dimensional Calculation**
  - **3-Dimensional Calculation**

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<tr>
<td>No. of Unknowns</td>
<td>$N$ (2-D) $N^2$ (3-D)</td>
<td>$N^2$ (2-D) $N^3$ (3-D)</td>
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<tr>
<td>Memory Requirement</td>
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<td>Time Steps</td>
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<tr>
<td></td>
<td>$N^3$ (2-D) $N^6$ (3-D)</td>
<td>$N^3$ (2-D) $N^4$ (3-D)</td>
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<tr>
<td>Computer Time</td>
<td>$N^2$ (2-D) $N^4$ (3-D)</td>
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Geometrical Optics (GO) - Overview

• Geometrical Optics (GO) is an approximate method for RCS calculation
  – Valid in the “optical” region (target size >> $\lambda$)

• Based upon ray tracing from the radar to “specular points” on the surface of the target
  – “Specular points” are those points, whose normal vector points back to the radar.

• The amount of reflected energy depends on the principal radii of curvature at the surface reflection point

• Geometrical optics (GO) RCS calculations are reasonably accurate to 10 – 15% for radii of curvature of 2 $\lambda$ to 3$\lambda$

• The GO approximation breaks down for flat plates, cylinders and other objects that have infinite radii of curvature; and at edges of these targets

$$\sigma = \pi \rho_1 \rho_2$$
Geometric Optics

- **Power Density Ratio**
  \[
  \frac{\langle \vec{S} \rangle_{SCAT}}{\langle \vec{S} \rangle_{INC}} = \frac{1}{A_S} = \frac{A_I}{A_S} = \frac{a^2}{4} \frac{d\Omega}{R^2 d\Omega}
  \]

- **Radar Cross Section of Sphere**
  \[
  4\pi R^2 \frac{\langle S_{SCAT} \rangle}{\langle S_{INC} \rangle} = 4\pi R^2 \frac{a^2}{4 R^2} = \pi a^2
  \]

- **Radar Cross Section of an Arbitrary Specular Point**
  \[
  \pi \rho_1 \rho_2
  \]
  - Where radii of curvature at specular point = \( \rho_1, \rho_2 \)
Single and Double Reflections

(Geometrical Optics Method)

- **RCS Calculation for Single Reflection**
  - Identify all specular points and add contributions
  - Phase calculated from distance to and from specular point
  - Local radii of curvature used to determine amplitude of backscatter

- **RCS Calculation for Double Reflection**
  - Identify all pairs of specular points
  - At each reflection use single reflection methodology to calculate amplitude and phase
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Physical Optics (PO) Overview

• Physical Optics (PO) is an approximate method for RCS calculation
  – Valid in the “optical” region (target size $>> \lambda$)

• Method - Physical Optics (PO) calculation
  – Modify the Stratton-Chu integral equation form of Maxwell’s Equations, assuming that the target is in the far field
  – Assume that the total fields, at any point, on the surface of the target are those that would be there if the target were flat
    Called “Tangent plane approximation”
  – Assume perfectly conducting target
  – Resulting equation for the scattered electric field may be readily calculated
  – RCS is easily calculated from the scattered electric field

• Physical Optics RCS calculations:
  – Give excellent results for normal (or nearly normal) incidence ($< 30^\circ$)
  – Poor results for shallow grazing angles and near surface edges
    e.g. leading and trailing edges of wings or edges of flat plates
Physical Optics

Tangent Plane Approximation

For an incident plane wave:

\[ \mathbf{J}_s (\mathbf{r}') = 2 \hat{n} \times \mathbf{H}_o e^{-i \mathbf{k} \cdot \mathbf{r}' \mathbf{r}} \]

Substituting this surface current yields (for the monostatic case):

\[ \mathbf{E}_s (\mathbf{r}) = -2i \omega \mu \frac{e^{ikr}}{4\pi r} \int \hat{r} \times \hat{r} \times \left( \hat{n} \times \mathbf{H}_o \right) e^{-2i \mathbf{k} \cdot \mathbf{r}' \mathbf{r}} d\mathbf{r}' \]
Normal and Oblique Incidence

- Physical Optics contribution adds constructively (in phase).
- For large plates, the edge contribution is a small part of the total current.
- Except near the edges, Physical Optics gives accurate results.

Except near the edges, Physical Optics gives accurate results.

- Fresnel Zones of alternating phase caused by phase delay across plate.
- In the backscatter direction, the Physical Optics contribution is predominantly cancelled.
- The most significant part of total current due to edge effects.
Monostatic RCS of a Square Plate

- 15 cm x 15 cm Plate
- 10.0 GHz
- HH Polarization

\[
\sigma_{\text{MAX}} = 4 \pi \frac{A^2}{\lambda^2}
\]

Measurement
Physical Optics (PO) Approximation

Aspect Angle (degrees)
Radar Cross Section (dBsm)
# Methods of Radar Cross Section Calculation

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Geometrical Theory of Diffraction (GTD) Overview

- Geometrical Theory of Diffraction (GTD) a ray tracing method of calculating the diffracted fields at surface edges / discontinuities
  - Assumption: When ray impinges on an edge, a cone (see Keller (1957) Cone below) of diffracted rays are generated
  - Half angle of cone is equal to the angle, $\beta$, between the edge and the incident ray.
    - In backscatter case the cone becomes a disk
  - Diffracted electric field proportional to “diffraction coefficients”, $X$ and $Y$ and a “divergence factor, $\Gamma$, and given by:

\[
|\vec{E}_{DIF}| = \frac{\Gamma e^{ik\phi} e^{i\pi/4}}{\sin \beta \sqrt{2\pi ks}} (X \mp Y)
\]

- Diffraction coefficients
  - $-$ when $\vec{E}_1$ parallel to edge
  - $+$ when $\vec{H}_1$ parallel to edge
- Divergence factor reduces amplitude as rays diverge from scattering point and accounts for curves edges

\[
\Gamma = \frac{\pi}{2} \left( \frac{1}{\sin \beta} \right)
\]
Geometrical Theory of Diffraction (GTD)

Ray Tracing (With Creeping Waves and Diffraction)

Cylinder

\[ \vec{E}_{\text{SCAT}} = \vec{E}_{\text{ER}} + \vec{E}_{\text{SR}} + \vec{E}_{\text{DIFF}} + \vec{E}_{\text{CW}} \]

Disadvantages
- Implementation difficult for complex targets
- Requires more accurate description than PTD

Plate

\[ \vec{E}_{\text{SCAT}} = \vec{E}_{\text{REFL}} + \vec{E}_{\text{EDGE}} + \vec{E}_{\text{COR}} \]

Advantages
- Easy to Understand
- Multiple Interactions

Endcap Reflection

Cylindrical surface

Side Reflection

Creeping Wave

Diffraction

Reflection

Corner Diffraction

Edge Diffraction

Endcap Diffraction

Cylinder

Plate
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Physical Theory of Diffraction (PTD) Overview

- **Approach:** Integrate surface current obtained from local tangent plane approximation (plus edge current)
- **Advantages:** Reduced computational requirements and applicable to arbitrary complex geometries
- **Disadvantages:** Neglects multiple interactions or shadowing

\[ J = \vec{J}_{PO} + \vec{J}_{DIF} \]
• In 1896, Sommerfeld developed a method to find the total scattered field for an infinite, perfectly conducting wedge.
• In 1957, Ufimtsev obtained the edge current contributions by subtracting the physical optics contributions from the total scattered field.
• The current for finite length structures may be obtained by truncating the edge current from that of the infinite structure.
Normal and Oblique Diffraction

**Diffraction Perpendicular to Edge**
- Constructive addition from edge current contribution along entire edge results in strong perpendicular backscatter
- Small contribution from corner edge current
- Perpendicular to edge, scattering is strong in all directions

**Oblique Diffraction**
- Edge current contribution interferes destructively in direction of backscatter
- For near grazing angles, corner current may be significant
- Strong scattering along “Keller Cone”
Trailing / Leading Edge Diffraction

- Negligible scattering at front edge – Electric field normal and continuous
- Traveling waves; above and below plate develop a relative phase delay.
- Required continuity of electric field at back edge causes induced edge current, and thus a diffracted electric field.

Trailing Edge Diffraction

- Tangential component of electric field equals zero along the conductor.
- Diffracted electric field is produced by current induced to cancel incident electric field.
- No diffraction at back edge because electric field is close to zero.
FD-TD Simulation of Scattering by Strip

Case 2

- Gaussian pulse plane wave incidence
- E-field polarization \( (E_y \text{ plotted}) \)
- Phenomena: leading edge diffraction

Courtesy of MIT Lincoln Laboratory
Used with permission
FD-TD Simulation of Scattering by Strip

Case 2

Courtesy of MIT Lincoln Laboratory
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FD-TD Simulation of Scattering by Strip

Case 3

- Gaussian pulse plane wave incidence
- H-field polarization ($H_y$ plotted)
- Phenomena: trailing edge diffraction

Courtesy of MIT Lincoln Laboratory
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FD-TD Simulation of Scattering by Strip

Case 3

Courtesy of MIT Lincoln Laboratory
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Monostatic RCS of a Square Plate

- 15 cm x 15 cm Plate
- 10.0 GHz
- HH Polarization

Radar Cross Section (dBsm)

Aspect Angle (degrees)

Measurement
Physical Optics (PO) Approximation
Physical Theory Of Diffraction (PTD)

15 cm x 15 cm Plate
10.0 GHz
HH Polarization

Courtesy of MIT Lincoln Laboratory
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Measured and Predicted RCS of JGAM

Johnson Generic Aircraft Model (JGAM) at RATSCAT Outdoor Measurement Facility

- VV polarization
- Elevation = 7°
- 9.67 GHz

RATSCAT Measurement
PTD Prediction

Tail Broadside Nose Broadside Tail

- End Cap
- Fuselage Specular
- Cone Specular
- Wing Leading Edge
- Wing Trailing Edge

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Radar Cross Section Calculation Methods

• Introduction
  – A look at the few simple problems

• RCS prediction
  – Exact Techniques
    Finite Difference- Finite Time Technique (FD-FT)
    Method of Moments (MOM)
  – Approximate Techniques
    Geometrical Optics (GO)
    Physical Optics (PO)
    Geometrical Theory of Diffraction (GTD)
    Physical Theory of Diffraction (PTD)

• Comparison of different methodologies
RCS Prediction Techniques Family Tree

**Exact Techniques**
- Limited Geometry
- All Phenomena

**Approximate Techniques**
- Limited Phenomena
- Computationally Speedy
- Valid for High Frequencies

**Classical Solutions**
- Few Geometries
- Rigorous, Exact
- Series Solutions

**Numerical Methods**
- Computationally Slow
- Low Frequency

**Hybrid Methods**
- MoM / UTD
- MoM / PO
- MoM / GO

**Surface Integral Techniques**
- Computationally Slow
- All Geometries
- Physical Optics (PO)
- Physical Theory of Diffraction (PTD)

**Ray Tracing Techniques**
- Computationally Slow
- All Geometries
- Geometrical Optics (GO)
- Geometrical Theory of Diffraction (GTD)
- Universal Theory of Diffraction (UTD)
- Shooting and Bouncing Rays (SBR)

**Integral Equation Techniques**
- Method of Moments (MoM)
- Other Integral Techniques

**Differential Equation Solutions**
- Finite Element
- Finite Difference-Time Domain (FT-TD)
- Finite Difference-Frequency Domain (FD-FD)
Comparison of Different RCS Calculation Techniques

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<td>Multiple Reflection Diffraction</td>
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<td>Advantages</td>
<td>Exact Visualization Aids Physical Insight</td>
<td>Exact</td>
<td>- Simple Formulation - Good Insight into Physical Phenomena</td>
<td>Easiest Computationally - Good Insight into Physical Phenomena</td>
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<tr>
<td>Limitations And/or Disadvantages</td>
<td>- Low Frequency Only - Complex Geometries Difficult - Single Incident Angle</td>
<td>- Low Frequency Only - Formulation Difficult (Materials) - Single Frequency</td>
<td>- High Frequency Only - Canonical Geometries Only - Caustics</td>
<td>- High Frequency Only - Many Phenomena Neglected</td>
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Corner Reflectors

- Give a large reflection, $\sigma$, over a wide range of angles
  - Used as test targets and for radar calibration
- Different shapes
  - Dihedral
  - Trihedral
    Square, triangular, and circular

Ray Trace for a Dihedral Corner Reflector (Side view)

RCS of Dihedral Corner Reflector (Broadside Incidence)

$$\sigma = \frac{4\pi A_{EF}^2}{\lambda^2}$$

$A_{EF} = \text{Area of projected aperture}$

On the incident ray

Physical Optics Model

Sailboat Based Circular Trihedral Corner Reflector

Courtesy of dalydaly
Summary

• Target RCS depends on its characteristics and the radar parameters
  – Target: size, shape, material, orientation
  – Radar: frequency, polarization, range, viewing angles, etc

• The target RCS is due to many different scattering centers
  – Structural, Propulsion, and Avionics

• Many RCS calculation tools are available
  – Take into account the many different electromagnetic scattering mechanisms present

• Measurements and predictions are synergistic
  – Measurements anchor predictions
  – Predictions validate measurements
References


Acknowledgement

- Dr. Robert T-I. Shin
- Dr. Robert K. Atkins
- Dr. Hsiu C. Han
- Dr. Audrey J. Dumanian
- Dr. Seth D. Kosowsky
Homework Problems

• From Skolnik (Reference 2)
  – Problems 2-10, 2-11, 2-12, and 2-13

• From Levanon (Reference 6)
  – Problems 2-1 and 2-5

• For an ellipsoid of revolution, (semi major axis, $a$, aligned with the x-axis, semi minor axis, $b$, aligned with the y axis, and axis of rotation is the x-axis; what are the radar cross sections (far field) looking down the x, y, and z axes, if the radar has wavelength $\lambda$ and $a >> \lambda$ and $b >> \lambda$?

• Extra credit: Solve the last problem assuming $a << \lambda$ and $b << \lambda$. 